Quick Assignment 1
Total: 100

CS 2500: Algorithms

Due Date: August 29, 2024 at 11.59 PM

Instructions

- Submit your solutions by the deadline specified above.
- Ensure that your work is your own.
- Write your answers clearly and show all your work.
- If you have any questions, ask during recitations or office hours.

Note

While Quick Assignments typically consist of 2-3 problems, this one is slightly longer as it also includes material from last week. To accommodate the additional problems, you will have 48 hours instead of the usual 24 hours to complete this assignment. However, each problem is designed to be concise, ensuring that the overall assignment remains manageable and "quick."

Problems

1. Show that Algorithm `MaxElement` (Lecture 3) for finding the maximum element in a finite sequence of integers has all the properties of an algorithm discussed in Lecture 1. [**10 points**]
2. Show that $7n^2 \in O(n^3)$. Is it also true that $n^3 \in O(7n^2)$? [**15 points**]
3. Let k be a positive integer. Show that $1^k + 2^k + \cdots + n^k$ is $O(n^{k+1})$. [**15 points**]
4. Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n^2 \cdot 2^n$ operations and the second algorithm uses exactly $n!$ operations. As n grows, which algorithm uses fewer operations? [**20 points**]
5. Show that $\log(x^2 + 1)$ and $\log_2 x$ are of the same order. [**15 points**]
6. Devise an algorithm that finds a mode in a list of non-decreasing integers. Analyze the worst-case time complexity of the algorithm you devised. (Recall that a list of integers is non-decreasing if each term is at least as large as the preceding term.) [**25 points**]