CS 2500: Algorithms
Missouri University of Science and Technology
Final Exam
Fall 2024

(***Maximum Points: 50***)
(***Time allowed: 90 minutes***)

This Paper consists of 5 printed pages and 1 blank page.

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

*Answer **five** questions from Part I and **four** questions from Part II.*

Questions are numbered using Arabic numerals $(1, 2, 3, \dots)$*. If the question contains subparts, then you are required to answer **every subpart** of that question. If you have any doubts, please ask the proctor for clarification.*

The intended points for questions or parts of questions are given in brackets [].

*You are required to clearly show all steps involved in solving each problem (**except for MCQs**). This includes clearly stating any assumptions made during the problem-solving process, providing step-by-step calculations or logical deductions, and demonstrating how you arrive at your final answer. Each step should be justified with appropriate reasoning, explanations, or theorems where applicable.*

Whenever a question specifically asks for an algorithm, it MUST be written in pseudocode format (for example, as shown in Algorithm 1 on Page 4). Failure to adhere to this will result in zero points for that part of the question.

Failure to provide clear workings, explanations, or logical reasoning may result in a deduction of points, even if the final answer is correct.

Plan your time wisely. Do not spend too much time on any one problem. Read through all of them first and attack them in the order that allows you to make the most progress.

Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.

Important: *Please ensure that you write (1) full name, (2) student ID, and (3) page number on **EVERY** answer sheet provided. Sheets may get separated during grading.*

ALL THE BEST!

PART I [10 Points]

*Answer **any five** questions.*

1. Consider three algorithms A, B, and C with respective time complexities:

$$t_A(n) = 7n, \quad t_B(n) = 7\log_{10} n, \quad t_C(n) = n^2$$

Suppose the input size is scaled from $n = 100$ to $n = 10,000$. Which of the following correctly compares the growth of the run times of the algorithms? [2]

   (a) Algorithm A grows faster than both B and C.

   (b) Algorithm B grows faster than both A and C.

   (c) Algorithm C grows faster than both A and B.

   (d) Algorithm A and Algorithm B grow at the same rate, and both grow faster than C.

2. A sorting technique is called stable if: [2]

   (a) It takes $O(n \log n)$ time

   (b) It maintains the relative order of occurrence of non-distinct elements

   (c) It uses divide and conquer paradigm

   (d) It takes $O(n)$ space

3. What is the base case in a recursive algorithm? [2]

   (a) The case that makes the recursion continue

   (b) The case that prevents infinite recursion by providing a stopping condition

   (c) The case that maximizes the recursion depth

   (d) The case that combines results of smaller sub-problems

4. Which of the following algorithms uses the divide-and-conquer approach? [2]

   (a) Dijkstra's algorithm

   (b) Bubble Sort

   (c) Merge Sort

   (d) Kruskal's algorithm

5. Which of the following is NOT a characteristic of problems solvable by dynamic programming? [2]

   (a) Overlapping sub-problems

   (b) Greedy choice property

(c) Optimal substructure

(d) Memorization of intermediate results

6. Which of the following is true for greedy algorithms? [2]

   (a) They always provide the optimal solution for all problems.

   (b) They make decisions that are globally optimal.

   (c) They make locally optimal choices to build a solution.

   (d) They are only applicable to graph problems.

7. Which of the following problems is typically solved using a greedy algorithm? [2]

   (a) Longest Common Subsequence

   (b) 0/1 Knapsack Problem

   (c) Huffman Encoding

   (d) All-Pairs Shortest Path

PART II [40 Points]

*Answer **any four** questions.*

1. You are a data analyst at Spotify. The company has collected a list of the genres of songs played by users over the past month. They want to identify the genre that was played most frequently to tailor their recommendations and promotional efforts. If multiple genres have the same highest play count, any one of them can be considered.

   (a) Design a recursive algorithm that finds the genre that appears most frequently in the list of song genres. [3]

   (b) Analyze the time complexity of your recursive algorithm by deriving its recurrence relation. [3]

   (c) Prove the correctness of your algorithm using mathematical induction. [4]

2. The English coinage before decimalization included half-crowns (30 pence), florins (24 pence), shillings (12 pence), sixpences (6 pence), threepences (3 pence), and pennies (1 pence). The greedy algorithm `MakeChange` selects the largest denomination that does not exceed the remaining amount to make change. Consider the following scenario:

   (a) Using the greedy algorithm `MakeChange`, find the set of coins used to make 48 pence. Show your work step by step. [2]

(b) Find a better solution for making 48 pence than the one obtained in part (a). Clearly explain why this solution is better in terms of the number of coins used. [4]

(c) Explain why the greedy algorithm fails to find the optimal solution in this case. Include a general condition or example where the greedy approach may fail for coin-changing problems. [4]

Algorithm 1 MakeChange Algorithm

Require: n: the amount to make change for
Ensure: S: the set of coins used to make change

```
C ← {30, 24, 12, 6, 3, 1}   ▷ Available coin denominations: half-crowns, florins, shillings, sixpences, threepences, and pennies
S ← ∅                       ▷ Initialize the solution set
s ← 0                       ▷ Sum of the coins in the solution
while s ≠ n do
    x ← Largest item in C such that s + x ≤ n
    if x does not exist then
        return "no solution found"
    end if
    S ← S ∪ {x}             ▷ Add the selected coin to the solution
    s ← s + x               ▷ Update the sum
end while
return S
```

3. A delivery truck has a maximum weight capacity of **100 units**, and you need to load it with valuable packages from a warehouse. Each package has a specific weight and value, as shown in the table below:

Package ID	Weight (W)	Value (V)
A	10	20
B	20	30
C	30	66
D	40	40
E	50	60

We want to maximize the profit while respecting the truck's weight limit.

(a) Calculate the value-to-weight ratio for each package. Arrange the packages in descending order of their value-to-weight ratio. [4]

(b) Determine the optimal selection of packages (include fractional selections if necessary) to maximize the total value loaded into the truck. Clearly show which fractions of packages are selected, if applicable. [4]

(c) Show all your calculations and clearly state the maximum profit that can be obtained with the optimal selection. [2]

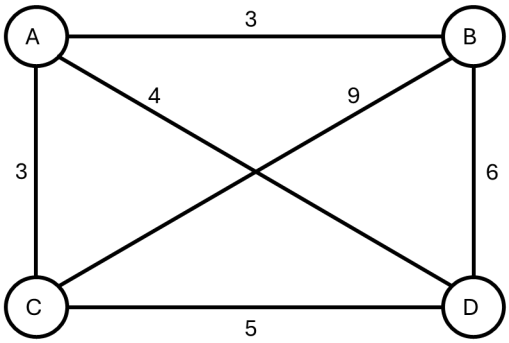
4. A communication company needs to efficiently transmit messages over a network using data compression techniques. You are tasked with constructing a Huffman code for the following set of characters and their probabilities of occurrence:

Character	Probability
A	0.35
B	0.1
C	0.2
D	0.2
E	0.15

   (a) Construct a Huffman code for the given characters. Show all steps involved in building the Huffman tree and assigning binary codes to each character. Place the smaller frequency node as the left child. [6]

   (b) Using the generated Huffman code, encode the message `ABCAD`. Show your encoding process clearly. [4]

5. You are tasked with designing a network for a small city. The city has five districts that need to be connected by roads. The network below shows the potential roads between districts, along with their construction costs (as edge weights):



   The city council has the following requirements:

   - All districts must be connected so that people can travel between any two districts.
   - The total cost of road construction must be minimized.
   - Roads must be selected in such a way that no loops (cycles) are formed in the network.

   (a) Identify the set of roads that should be selected to meet the requirements using Kruskal's Algorithm. Clearly explain your process for selecting the roads, including the order in which roads are considered. [5]

   (b) Draw the resulting network (graph) showing the districts as nodes and the selected roads as edges, labeled with their costs. [3]

(c) Calculate the total cost of the selected roads and confirm that it meets the council's requirement for minimizing cost. [2]

6. A charity organization has received a set of donations in specific amounts and wants to allocate the donations to fund a specific project that requires exactly \$50 to complete. The organization needs your help to determine whether it is possible to select a subset of donations that add up to the required amount.

   Here is the list of donation amounts (in \$):

   $$\{10, 20, 15, 5, 25, 30, 35\}$$

   (a) Using a step-by-step approach, determine if there exists a subset of the given donation amounts that adds up to the target value \$50. Show your work clearly. [4]

   (b) If a valid subset exists, list the subset. If no such subset exists, explain why. [2]

   (c) Describe the specific method or algorithm you used to solve this problem. Clearly explain how it determines whether a subset with the required sum exists, including its key steps. Additionally, discuss the efficiency of your method and any limitations it might have when applied to larger datasets or higher target values. [4]